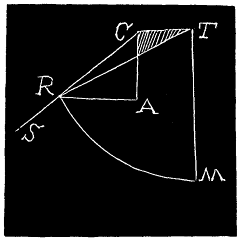
79. **Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.**

In latitude $42^\circ\ 30'$ N.$=\lambda$, a tree 100 feet long$=\alpha$, leans in the direction S. 60° W.$=\beta$, with an angle of elevation with the level ground, of $30^\circ=\gamma$. The sun's declination being $1^\circ\ 36'\ 24''$ N.$=\delta$, in what direction will the shadow of the tree point, when the sun is on the meridian ?

I. **Solution by the PROPOSER.**

Let $TR=100=a$, be the tree, the top at R, TM a south line from T.

Draw a radii from R to A, and AT will be a sub-projection of RT, on the ground, the triangles TAR and CAR being vertical. Angles MTR, and $MTA=60^\circ$ each$=\beta$, the triangle CAT being horizontal. Angle $ATR=30^\circ=\gamma$, being the elevation of TR.



The shadow of R made by the sun at S must be cast north of A, and appear on the ground at C, and all the termini of shadows of all other points from R to T must be cast and visible on the line CT, as the sun is practically south of all points on the tree.

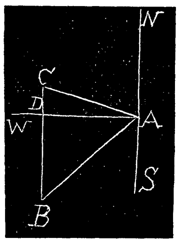
Angle $ACR=90^\circ-\lambda+\delta=49^\circ\ 6'\ 24''=c=$the sun's meridian altitude. $AR=a\sin\gamma=50$ feet$=e$, $AC=e\cot c$ $=43.3013$ feet$=\delta$, $AT=a\cos\gamma=86.6025$ feet$=b$.

Whatever angle CT makes with TM, counted from the south westward, will give the direction of the shadow.

CA must be parallel with TM. $\therefore$ Angle $CAT=ATM=60^\circ$. Then in triangle ACT we have given AC and AT, and the included angle of 60°, to find the angle CTA, which, by Plane Trigonometry, is easily found to be 30°, ACT becoming in this particular case a right angle. Adding the angles CTA and ATM gives $MTC=90^\circ$, and therefore the shadow points west.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

Let NS be a north and south line on the ground, AB the projection of the tree upon the ground, AW the west line on the ground, BL perpendicular to AW, AC the shadow, $\angle NAC=\alpha$, and $\angle BAS=\beta$. $AB=a\cos\gamma$, $BD=\alpha\cos\gamma\cos\beta$.



Since $90^\circ-\lambda+\delta$ is the meridianal altitude we have $BC=\alpha\sin\gamma\tan(\lambda-\delta)$.

$\therefore\ CD=\alpha\sin\gamma\tan(\lambda-\delta)-\alpha\cos\gamma\cos\beta$.

$$\therefore\ \cot x=\frac{CD}{AD}=\frac{\alpha\sin\gamma\tan(\lambda-\delta)-\alpha\cos\gamma\cos\beta}{\alpha\cos\gamma\sin\beta}.$$

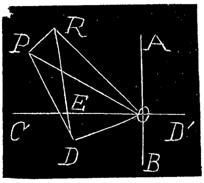
$$\therefore\ \cot x=\frac{\tan\gamma\tan(\lambda-\delta)}{\sin\beta}-\cot\beta.$$

Putting $\dfrac{\tan\gamma\tan(\lambda-\delta)}{\sin\beta}=\cot\varepsilon$, we get $\cot x=\dfrac{\sin(\beta-\varepsilon)}{\sin\beta\sin\varepsilon}$.

For the given numerical values, $\gamma=30°$, $\beta=60°$, $\lambda-\delta=40°\ 53'\ 36''$, we get $\varepsilon=60°$. $\therefore\ \beta-\varepsilon=60°$. $\therefore\ \cot x=0$. $\therefore\ x=90$. Consequently the shadow falls due west.

III. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let $OP=100$ feet$=a$ be the tree, AB the meridian, CD' the parallel of latitude, OD the projection of the tree on the plane, OR the shadow, $\angle POD=30°=\gamma$, $\angle DOB=60°=\beta$, $\angle PRD=$sun's altitude$=\frac{1}{2}\pi-(\lambda-\delta)$, $\angle RDO=\beta$, $\angle DOE=\frac{1}{2}\pi-\beta$, $\angle PDO=\angle PDR=\angle DEO=\frac{1}{2}\pi$.



$\therefore\ PD=a\sin\gamma=50$ feet, $DO=a\cos\gamma=50\sqrt{3}$.

$\therefore\ DO=86.60$ feet, $DE=DO\cos\beta=43.30$ feet, $DR=PD\tan(\lambda-\delta)=a\sin\gamma\tan(\lambda-\delta)=43.30$ feet. $\therefore\ DE=DR$ and E and R coincide.

$\therefore$ The shadow is due west.

Also solved by *EDMUND FISH* and *A. H. BELL*, and *H. C. WHITAKER*.

80. **Proposed by the late SYLVESTER ROBINS, North Branch, N. J.**

Exhibit ten initials in that infinite series of integral, rational rhombuses wherein the area of every term is one unit less than the square of its side.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let $2a$ and $2b$=the respective diagonals of a rhombus. As the diagonals of a rhombus bisect each other at right angles, we then have, side of rhombus$=\sqrt{(a^2+b^2)}$, and area$=2ab$.

$\therefore$ From condition of problem, $2ab=a^2+b^2-1$, or $a^2-2ab+b^2=1$.

Whence $a-b=\pm1$.

$\therefore$ *The side of rhombus must be the hypothenuse of a right triangle whose legs are consecutive integers.*

Several methods for finding successive right triangles of this kind are given in THE AMERICAN MATHEMATICAL MONTHLY, Vol. IV, No. 1, pages 24—27.

Whence we find, for the first ten integral, rational rhombuses, the respective—

Diagonals,		*Sides,*	and *Areas.*
$2a$	$2b$	$\sqrt{(a^2+b^2)}$	$2ab=a^2+b^2-1$
8	6	5	24
42	40	29	840
240	238	169	28560
1394	1392	985	970224
8120	8118	5741	32959080
47322	47320	33461	1119638520
275808	275806	195025	38034750624
1607522	1607520	1136689	1292061882720
9369320	9369318	6625109	43892069261880
54608394	54608392	38613965	1491038293021224

The general formula for finding sides is $6S_{n-1}-S_{n-2}=S_n$.

Also solved by *A. H. BELL*, *CHAS. C. CROSS*, and *G. B. M. ZERR*.